Practice Exam 1 Solutions

① (a) $P = P(t) = 30{,}700 + 850t$

(b) ~~PCC~~ In 2020, 10 years after 2010

$$P(10) = 39{,}200$$

(c) $45{,}000 = 30{,}700 + 850t$

$14{,}300 = 850t$

$t = 16.82$ In the year 2026.

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② (a) Company A: $C = 40 + 0.15m$ (m = miles)

Company B: $C = 50 + 0.10m$

(b) Which company is cheaper depends on # of miles driven per day.

$40 + 0.15m = 50 + 0.10m$

$10 = 0.05m$

$m = 200$

If you drive less than 200 miles, A is cheper.

If you drive more than 200 miles, B is cheaper.

If you drive 200 miles they will cost the same.

(3) An increase from 5 to 10 is 100% increase and an increase from 30 to 50 is 66.67% incease

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(4) Let $f(t) = $ # of US billionaires in year t.

(a) $f(1985) = 13$; $f(1990) = 99$

(b) Ave. yearly increase $= \dfrac{f(1990) - f(1985)}{1990 - 1985} = \dfrac{99-13}{5} = 17.2$

(c) $f(t) = 17.2t + b$ (solve for b)

$13 = f(1985) = 17.2(1985) + b$

so $b = -34{,}129$

$$\boxed{f(t) = 17.2t - 34{,}129}$$

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(5) $P = R - C = 12x - (500 + 6x) = 6x - 500$ (x T-shirts sold)

Need to sell $83\frac{1}{3}$ or 84 shirts before profit is ≥ 0 so 84 shirts is how many to break even.

⑥ ⓐ $\dfrac{20{,}000 - 100{,}000}{20 - 0} = -4{,}000.$

So $\quad V = V(t) = 100{,}000 - 4{,}000t.$

(b) $V(5) = \$80{,}000$ in 2015

(c) The vertical intercept $(0, 100{,}000)$
tells us the bus is worth \$100,000 in 2010.

The horizontal intercept $(25, 0)$
tells us the bus is worthless in 2035.

(d) The domain of the fcn V is $[0, 25]$.

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⑦ Demand: $q = 120{,}000 - 500p$ Supply: $q = 1000p$.

(a) At $p = \$100$,
consumers will buy 70,000 units
and producers will supply 100,000 units.
Market will push prices down

(b)
$$120{,}000 - 500p = 1000p$$
$$p = \frac{120{,}000}{1500} = \$80.$$

(8) $P(8) = 10{,}000\, e^{.03 \cdot 8} = \$12{,}712.49$

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(9) $\$8000 = P(5) = P_0\, e^{.04 \cdot 5}$

$$P_0 = \frac{8000}{e^{.2}} = \$6{,}549.85$$

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(10) $\$20{,}000 = P(t) = 12{,}000\,(1.08)^t$

$$(1.08)^t = \frac{5}{3}$$

$$t\ln(1.08) = \ln(5/3)$$

$$t = \frac{\ln(5/3)}{\ln(1.08)} \approx 6.64 \text{ years}$$

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(11) The worker would prefer to receive \$2000 now so that in 1 year all \$2000 gains the 5% interest.

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(12) $2 = (1.07)^t$

$$\ln 2 = t\ln(1.07)$$

$$t = \frac{\ln 2}{\ln(1.07)} \approx 10.25 \text{ years}$$

(13) (a) $A = A(t) = 100\,e^{-.17t}$

(b) $\frac{1}{2} = e^{-.17t}$

$\ln(\frac{1}{2}) = -.17t$

$t = \frac{\ln(\frac{1}{2})}{-.17} \approx 4.08$ hours

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(14) $30{,}000 = 20{,}000\,a^5$

$\frac{3}{2} = a^5 \Rightarrow a = \sqrt[5]{3/2} \approx 1.084$ 8.4% annual increase

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(15) $2.7 = 5e^{-0.15t}$

$.54 = e^{-0.15t}$

$\ln(.54) = -0.15t \Rightarrow t = \frac{\ln(.54)}{-0.15} \approx 4.11$ years

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(16) Let $P(t)$ (in millions) be population of US t years after 2000.

(a) $308.7 = P(10) = 281.4\,e^{k10}$

$1.097 = e^{k10}$

$\ln(1.097) = k10 \Rightarrow k = \frac{\ln(1.097)}{10} \approx 0.00926$

So $350 = P(t) = 281.4e^{.00926 \cdot t}$

$1.244 = e^{.00926 \cdot t}$

$\ln(1.244) = .00926 \cdot t \Rightarrow t = \frac{\ln(1.244)}{0.00926} \approx 23.56$ years. So during 2023.

(b) $P(20) = 281.4\,e^{.00926 \cdot 20} = 338.7$ million.

(17) $f(x)=x^2$, $g(x)=\frac{1}{x}$, $h(x)=\sqrt{x-4}$, $l(x)=3x+2$

(a) $f\circ l(x)=f(l(x))=(3x+2)^2$

(b) $g(f(x))=\frac{1}{x^2}$

(c) $h(g(1/8))=h(8)=\sqrt{8-4}=2$

(d) $l(g(3))=l(\frac{1}{3})=3(\frac{1}{3})+2=3$

(e) $g\circ h(x)=g(h(x))=\frac{1}{\sqrt{x-4}}$

(f) $l(h(x+4))=l(\sqrt{x})=3\sqrt{x}+2$

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(18) (a) $C=12\ln u$ where $u=q^3+1$.

(b) $P=16e^u$ where $u=-0.6t$.

(c) $y=u^6$ where $u=5t^2-2$.

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(19) $f(x)=e^x$ (a) (i) $\frac{e^{0.1}-e^0}{0.1}=1.05$

(ii) $\frac{e^{0.01}-e^0}{0.01}=1.005$

(iii) $\frac{e^{0.001}-e^0}{0.001}=1.0005$

(b) $\frac{f(0+h)-f(0)}{h}=\frac{e^h-e^0}{h}$ is approaching 1

as h approaches 0.

So we would guess that

$f'(0)=1$.

20 $f(x)=x^3$ (a) (i) $\dfrac{(2.1)^3-2^3}{.1}=12.61$

(ii) $\dfrac{(2.01)^3-2^3}{.01}=12.06$

(iii) $\dfrac{(2.001)^3-2^3}{.001}=12.006$

(b) $\dfrac{f(2+h)-f(2)}{h}=\dfrac{(2+h)^3-2^3}{h}$ approaches 12

as h approaches 0.

So we would guess $f'(2)=12$.

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21 $(7,3)$ indicates that $f(7)=3$.

$f'(7)$ is given by the slope of the tangent line which is given by $\dfrac{\Delta y}{\Delta x}=\dfrac{3.8-3}{7.2-7}=\dfrac{.8}{.2}=4$.

So $f'(7)=4$.

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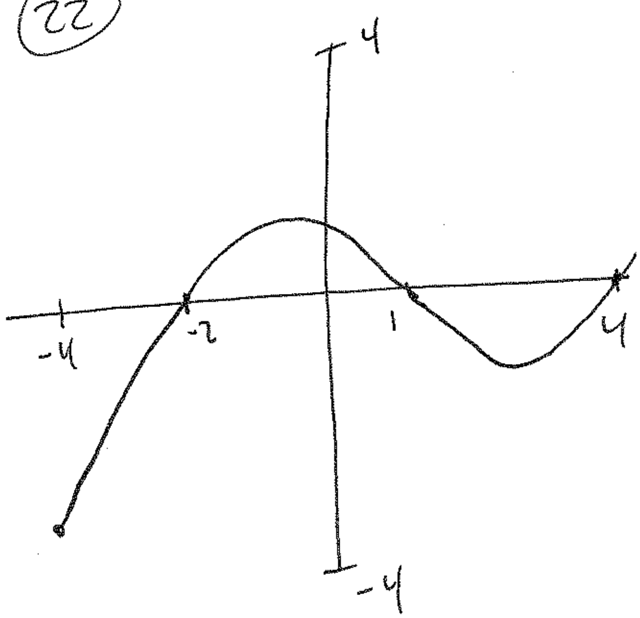

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